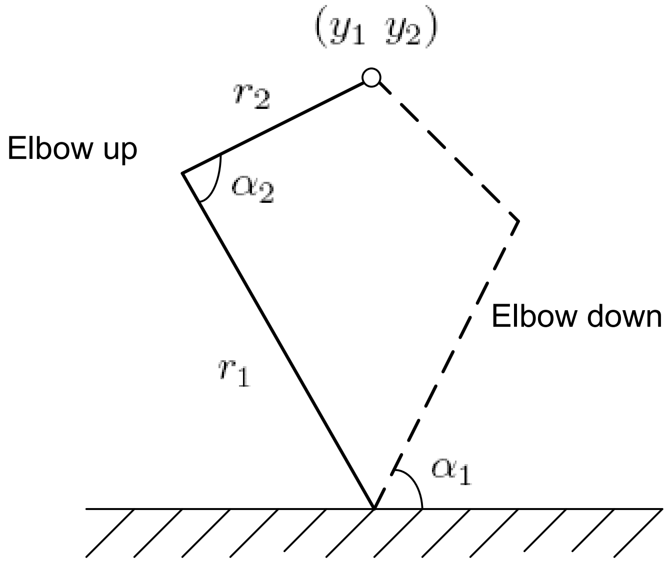


Fig. 1. Two link robot arm illustrating how the Cartesian coordinates (y_1, y_2) of the end effector is mapped to the given angles (α_1, α_2).

I. Tracking a Robot Arm

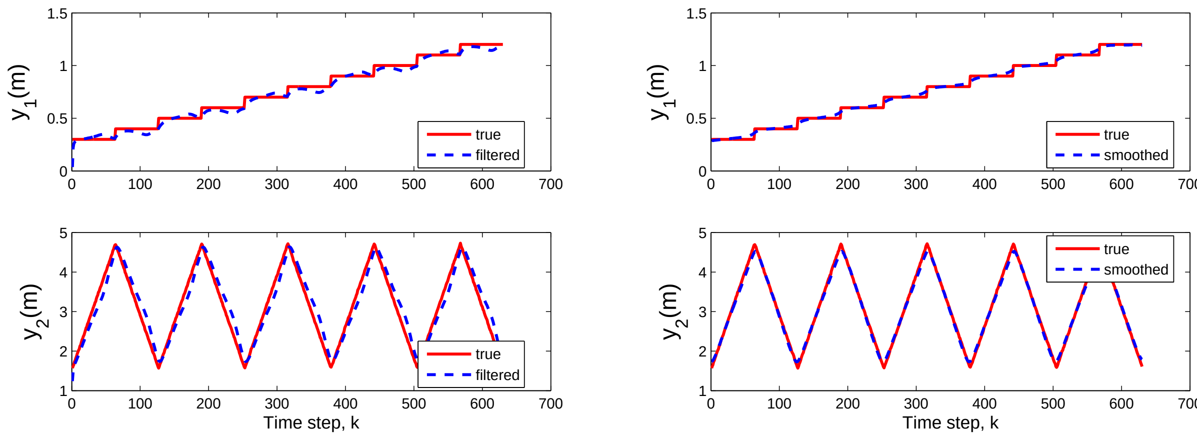
This article presents a simple example illustrating the power of the fixed-interval Cubature Kalman Smoother (CKS) over the Cubature Kalman Filter (CKF). Consider the kinematics of a two-link robot arm (see Fig. 1). Given the angles (α_1, α_2), the end effector position of the robot arm can be described in the Cartesian coordinate as follows:

$$\begin{aligned} y_1 &= r_1 \cos(\alpha_1) - r_2 \cos(\alpha_1 + \alpha_2) \\ y_2 &= r_1 \sin(\alpha_1) - r_2 \sin(\alpha_1 + \alpha_2), \end{aligned}$$

where $r1 = 0.8$ and $r2 = 0.2$ are the lengths of the two links; $\alpha_1 \in [0.3, 1.2]$ and $\alpha_2 \in [\pi/2, 3\pi/2]$ are the joint angles confined to a specific region. The solid and dashed lines in Fig. 1 show the 'elbow up' and 'elbow down' situations, respectively. The mapping from (α_1, α_2) to (y_1, y_2) is called the *forward kinematic*, whereas the *inverse kinematic* refers to the mapping from (y_1, y_2) to (α_1, α_2). The inverse kinematic is not a one-to-one mapping and thus its solution is not unique.

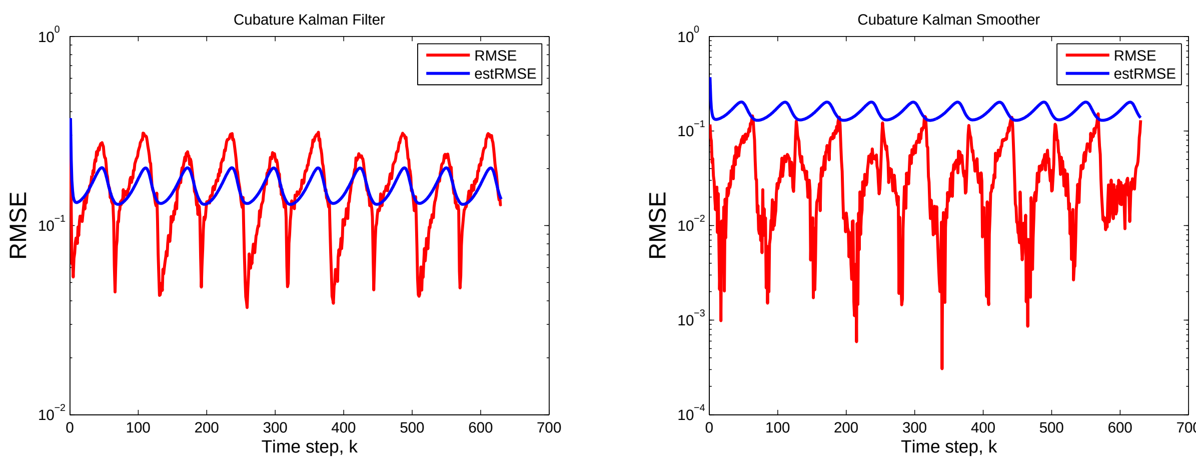
Let the state vector $\mathbf{x}$ be $\mathbf{x} = [\alpha_1\ \alpha_2]^T$ and the measurement vector $\mathbf{y}$ be $\mathbf{y} = [y_1\ y_2]^T$. The state-space model of the the given inverse kinematic problem can now be written as:

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{x}_k + \mathbf{w}_k \\ \mathbf{y}_k &= \begin{pmatrix} \cos(\alpha_{1,k}) & -\cos(\alpha_{1,k} + \alpha_{2,k}) \\ \sin(\alpha_{1,k}) & -\sin(\alpha_{1,k} + \alpha_{2,k}) \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \end{pmatrix} + \mathbf{v}_k \end{aligned}$$



(a) Cubature Kalman Filter (CKF) (b) Cubature Kalman Smoother (CKS)

Fig. 2. Tracking results (True trajectory- Solid line, Estimated Trajectory- Dotted line)



(a) Cubature Kalman Filter (CKF) (b) Cubature Kalman Smoother (CKS)

Fig. 3. Ensemble averaged (over 50 runs) root mean-squared error (RMSE) results (true rmse- red line, estimated rmse- blue)

We assume the state equation to follow a random-walk model perturbed by white Gaussian noise $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \mathrm{diag}[0.01, 0.1])$. The measurement equation is nonlinear with measurement noise $\mathbf{v} \in \mathcal{N}(0, 0.005\mathbf{I})$, where $\mathbf{I}$ is the identity matrix of appropriate dimension.

As can be seen from Fig. 2, α_1 is a slowly increasing process with periodic random walk whereas α_2 is a periodic, fast, and linearly-increasing/decreasing process. From Figs. 3(a) and 3(b), we see that the root mean square error of the CKS is less than that of the CKF as expected. Moreover, the CKS is more consistent than the CKF because the smoother estimated root mean square error is higher than the true root mean square error (Please find more about nonlinear Bayesian filtering at http://grads.ece.mcmaster.ca/ aienkaran/).